

AD

**TECHNICAL REPORT
NATICK/TR-82/009**

MATHEMATICAL MODELING OF THE BIAXIAL STRESS - STRAIN BEHAVIOR OF FABRICS

by

Earl C. Steeves

March 1982

**APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.**

**UNITED STATES ARMY
NATICK RESEARCH and DEVELOPMENT LABORATORIES
NATICK, MASSACHUSETTS 01760**



Aero-Mechanical Engineering Laboratory

Approved for public release; distribution unlimited.

Citation of trade names in this report does not constitute an official indorsement or approval of the use of such items.

Destroy this report when no longer needed. Do not return it to the originator.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NATICK/TR-82/009	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MATHEMATICAL MODELING OF THE BIAXIAL STRESS-STRAIN BEHAVIOR OF FABRICS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER NATICK/TR-82/009
7. AUTHOR(s) Earl C. Steeves		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Natick Research and Development Laboratories ATTN: DRDNA-UE Natick, MA 01760		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61101A 1T16110191A07118
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE March 1982
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
STRESS STRAIN FABRIC(S) BIAXIAL	MODEL MATHEMATICAL TENTS TENTAGE	FABRIC STRUCTURES STRUCTURAL BEHAVIOR DATA
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
<p>The development and increased use of computational structural design programs for fabric structures such as tents demands a description of the fabric stress-strain behavior in a form suitable for use on the computer. Here some work is described on the development of mathematical models to fill that need. The technique used is nonlinear least squares fitting of experimental data to chosen functions which treat the biaxial strain as the dependent variables and the biaxial stresses as the independent variables. The experimental results used are from</p>		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. ABSTRACT (continued)

biaxial stress tests in which the ratio of the stresses in the two orthogonal directions is maintained as a constant. Five sets of data, in each of which this constant has a different value, are used in the least squares problem. The results show a satisfactory fitting of the data from one fabric with stress-strain behavior that is typical of fabrics used in tentage. These results were obtained with three different power law functions of varying complexity. The most suitable of these functions depends on the particular use to be made of the results.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

TABLE OF CONTENTS

	Page
List of Illustrations	2
Introduction	3
Statement of the General Problem	4
Model Development	6
Concluding Remarks	23
Appendices:	
Appendix A. Biaxial Stress-Strain Data	27
Appendix B. Procedure for Execution of the Modeling Code	39

LIST OF ILLUSTRATIONS

Page

Figure

1	Coordinate System and Nomenclature	5
2	Stress-Strain Data Set FD21-2	7
3	Stress-Strain Data Set FD21-3	8
4	Behavior of Model 4 Based on Data Set FD21-2 and Comparison with Data	11
5	Behavior of Model 4 Based on Data Set DF21-3 and Comparison with Data	12
6	Behavior of Model 5 Based on Data Set FD21-2 and Comparison with Data	15
7	Behavior of Model 5 Based on Data Set FD21-3 and Comparison with Data	16
8	Behavior of Model 11 Based on Data Set DF21-2 and Comparison with Data	18
9	Behavior of Model 11 Based on Data Set FD21-3 and Comparison with Data	19
10	Behavior of Variable Exponents of Model 11	21
11	Behavior of the Reduced Model 11 Based on Data Set FD21-2 and Comparison with Data	25

Table

1	Parameter Estimates for Model 4	9
2	Parameter Estimates for Model 5	14
3	Parameter Estimates for Model 11	17
4	Parameter Estimates for the Reduced Model 11 and Data Set FD21-2	24

MATHEMATICAL MODELING OF THE BIAXIAL STRESS-STRAIN BEHAVIOR OF FABRICS

INTRODUCTION

In recent years advances have been made in the design analysis capability for fabric structures, see for example references 1 and 2 and, as would be expected, these analysis techniques require the specification of the stress-strain behavior of the fabric. Data describing this behavior, which is nonlinear, are scarce, and those available are in graphical and tabular form which are not suitable forms for computational purposes. To remedy this situation, a biaxial and shear stress-strain testing apparatus was developed and is described in reference 3. This apparatus provides stress-strain data on fabrics, and it is the purpose of this report to describe work on a method for putting this data in a form convenient for use with structural analysis computer codes.

In the present work only direct biaxial stress states are considered; shear deformation is not included. The analysis codes in which the results are to be used treat the fabric as an elastic membrane, so we do not include such effects as creep, hysteresis, or other nonconservative behavior which is certainly present. Models are constructed using specific sets of stress-strain data made up of a number of instantaneous values from a continuously run loading test.

The modeling method used is nonlinear curve fitting or least squares in which the strains are the dependent variables and the stresses are the independent variables. The nonlinear least squares problems are solved using the NL2SOL program described in reference 4. Power law functions of various forms are used in the modeling. A power law stress-strain law for fabrics was used in reference 2 and the models treated here are generalizations of that model.

1. Earl C. Steeves; The Structural Behavior of Pressure Stabilized Arches. US Army Natick Research & Development Command, Technical Report NATICK/TR-78/018. 1978 (AD-A063263)
2. Paul J. Remington, John C. O'Callahan and Richard Madden; Analysis of Stresses and Deflections in Frame Supported Tents. US Army Natick Laboratories, Technical Report 75-31. 1974 (AD A002 072)
3. Constantin J. Monego and Malcolm N. Pilsworth, Jr.; Development of an Apparatus for Biaxial and Shear Stress-Strain Testing of Fabrics and Films. US Army Natick Research & Development Laboratories, Technical Report NATICK/TR-80/028. 1980 (AD A094270)
4. J. E. Dennis, D. M. Gay, and R. E. Welsck; An Adaptive Nonlinear Least-Squares Algorithm. National Bureau of Economic Research, Inc., NBER Working Paper No. 196. 1977

STATEMENT OF THE GENERAL PROBLEM

We seek a model of the stress-strain behavior of fabrics and consider the case in which direct stresses are applied along the directions parallel to the warp and fill yarns of the fabric, and no shear stress is applied. As a result, these directions are the principal stress directions, and the strain state can be expressed in terms of the stresses as

$$\begin{aligned} E_1 &= E_1(T_1, T_2) \\ E_2 &= E_2(T_1, T_2) \end{aligned} \tag{1}$$

Where E_i and T_i are the strain and stress components and the coordinates X_1, X_2 are the fill and warp directions as shown in Figure 1. What we seek are functions E_1 and E_2 which describe experimental data. These data consist of sets of values of stress and strain; $(T_1, T_2, E_1, E_2)_{i,j}, (i=1, \dots, I) \text{ and } j=1, \dots, J)$. The test format from which these data are obtained has the ratio of the stresses $T_2/T_1 = \alpha$, a constant, so each value of j denotes a sequence of data for a given value of α . In this report a complete set of data will have $J = 5$ and α taking on the values $1/6, 1/2, 1, 2, 6$. As the subscript i increases for a fixed value of j , we have successive data points as the stress increases.

Having such a set of data we choose a specific functional form for E_1 and E_2 and determine the parameters of the function to best fit the data. For example, if a power law function is chosen we use the data to determine the magnitude coefficients and the exponents. Since, as in this example, parameters like the exponents must be determined, we have a nonlinear least squares problem to solve, and this is accomplished with the software package known as NL2SOL described in reference 4.

Having this functional representation of the stress-strain behavior is far more convenient than the tabular data because the functional form can be coded directly into structural analysis computer codes, while the tabular data would require such codes to have access to the data and an interpolation routine. In the structural analysis code described in reference 2, the functional form is not used directly but equation (1) is converted to a linear incremental form by expanding in a Taylor Series and keeping first order terms to give

$$\begin{aligned} \Delta E_1 &= \frac{\partial E_1}{\partial T_1} \Delta T_1 + \frac{\partial E_1}{\partial T_2} \Delta T_2 \\ \Delta E_2 &= \frac{\partial E_2}{\partial T_1} \Delta T_1 + \frac{\partial E_2}{\partial T_2} \Delta T_2 \end{aligned} \tag{2}$$

Most structural analysis codes rely on the symmetry of the force-displacement equation for their storage during the solution process. The symmetry of these equations depends on the symmetry of the stress-strain law which in the case of (2) requires that

$$\frac{\partial E_1}{\partial T_2} = \frac{\partial E_2}{\partial T_1} \tag{3}$$

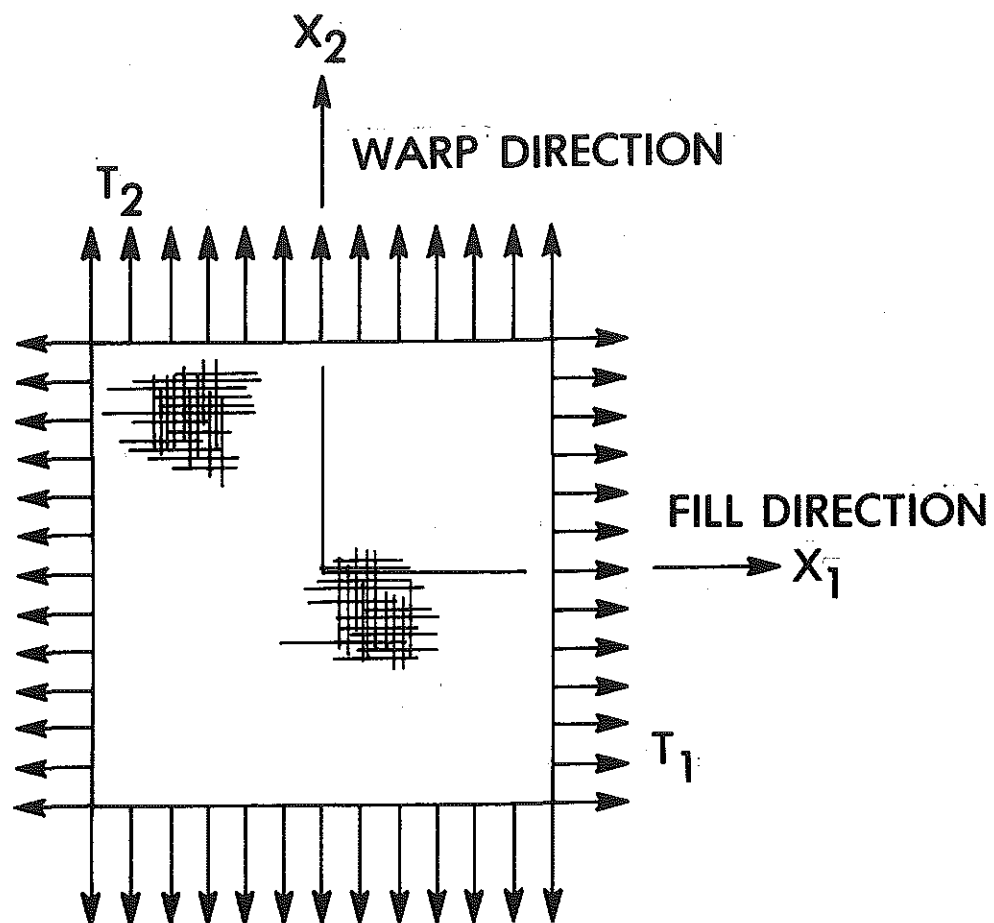


Figure 1. Coordinate system and nomenclature.

Thus, this is a desirable property of functions chosen to model the stress-strain behavior. It should be noted that this symmetry condition does not impose the constraint that the material be isotropic since

$$\frac{\partial E_1}{\partial T_1} \neq \frac{\partial E_2}{\partial T_2}$$

Use of the technique of nonlinear least squares for model construction requires some physical data to model. In this work we used data from a polyurethane-coated nylon fabric with a base fabric weight of 41 g/m² and a coated weight of 112 g/m². This fabric has a plain weave with 72 ends/cm and 44 picks/cm. The warp crimp of the fabric is 0.7% and the fill crimp is 2.3%. Two sets of biaxial stress-strain data for this fabric were used and they are shown graphically in Figures 2 and 3 and in numerical form in Appendix A. These two sets of data show generally the same behavior although data set FD21-2 appears somewhat more regular with respect to changes in stiffness with stress ratio. There are some marked differences between the data sets at low stress levels. For example, with the stress ratio at 1/6 data set FD21-3 shows that the fill direction experiences a contraction while data set FD21-2 shows only elongation. A similar difference is observed for the warp direction with the stress ratio at 2/1. The reason for these differences is not understood although we presently believe it is related to the test start-up procedure and the setting of the zero stress state in that procedure. Since the crimp interchange deformation of the fabric takes place at low stress levels, it can be markedly affected by setting the zero stress state. As will be shown later, these differences have a significant impact on the resulting model.

In the development of models, we looked at a number of classes of functions including polynomial, transcendental, and power law functions. The most satisfactory results were obtained with the power law functions, and we now proceed to describe some of these specific models and give some results.

MODEL DEVELOPMENT

The first of these models that we describe is a power law model which we call model 4 and has the form

$$\begin{aligned} E_1 &= C_{11} T_1^{P_1} + C_{12} T_1^{P_2} T_2^{P_3 + 1} \\ E_2 &= C_{12} \frac{P_3 + 1}{P_2 + 1} T_1^{P_2 + 1} T_2^{P_3} + C_{22} T_2^{P_4} \end{aligned} \quad (4)$$

This model, with seven parameters to be determined by least squares, has the symmetry property given by equation (3), and reduces to a linear model with $P_2 = P_3 = 0$ and $P_1 = P_4 = 1.0$. The linear model can be made isotropic by taking $C_{11} = C_{22}$.

The models obtained from the nonlinear least squares fitting of these functions to each of the data sets are given in Table 1 where we present the estimates of the model parameters and the standard deviations associated with these estimates. It is easily seen that the models for the two data sets are quite different and this merely reflects the difference in the data

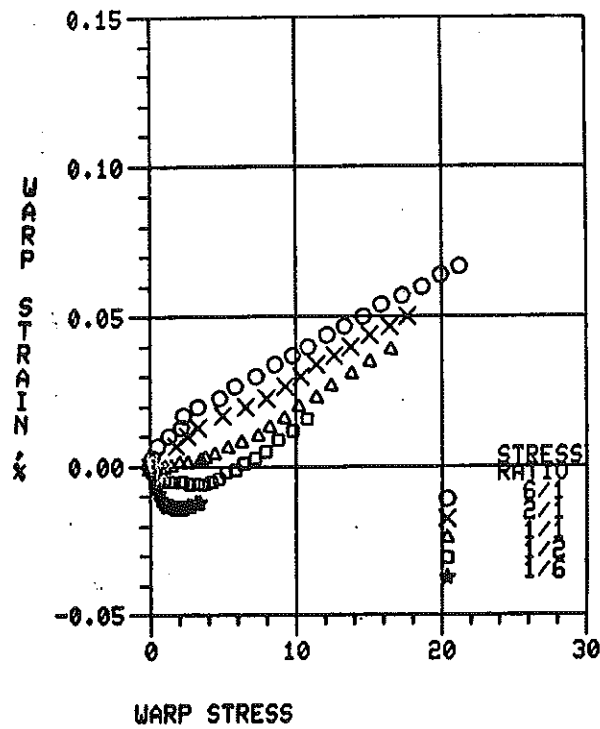
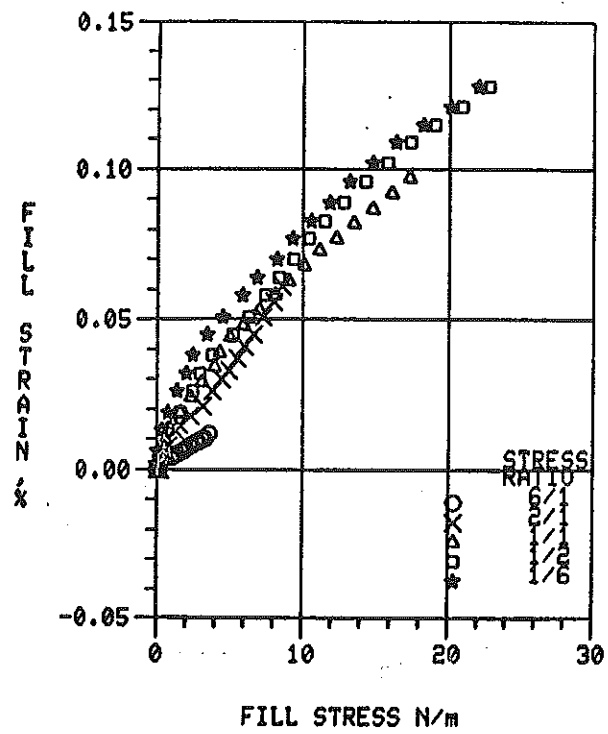


Figure 2. Stress-strain data for set FD21-2.

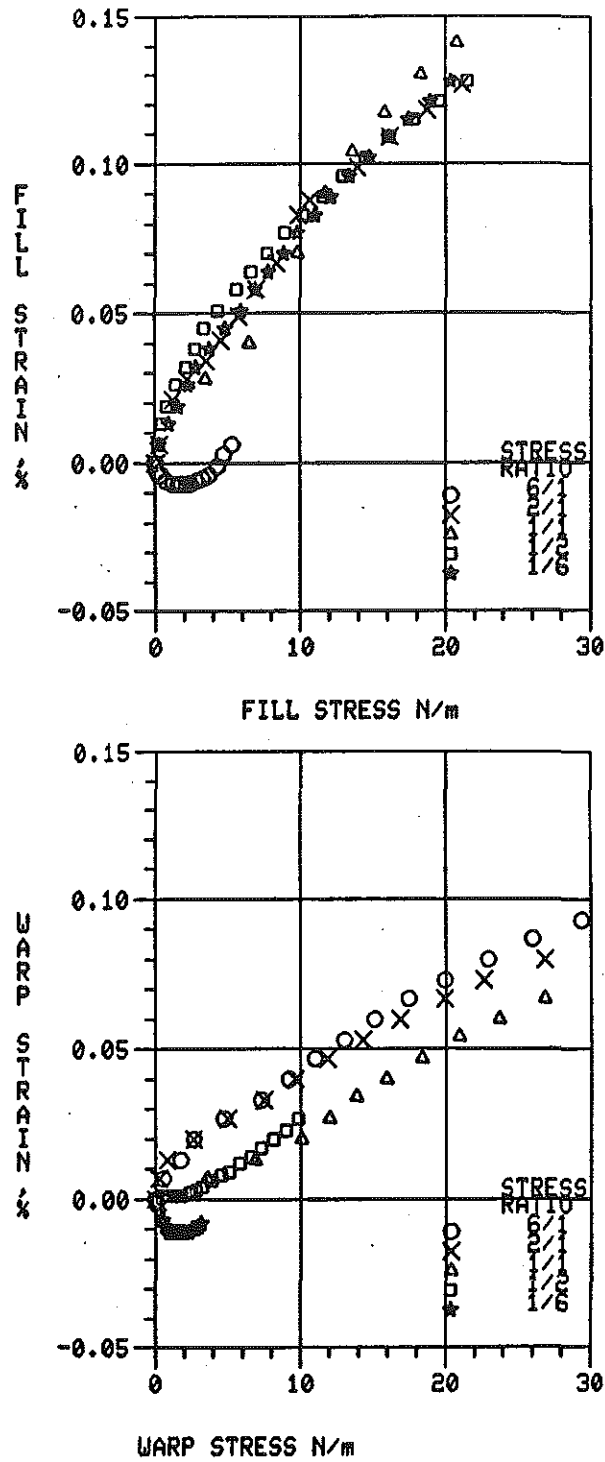


Figure 3. Stress-strain data for set FD21-3.

Table 1. Parameter Estimates for Model 4

Parameter	Data Set FD21-2		Data Set FD21-3	
	Parameter Estimate	Standard Deviation X 10^{+2}	Parameter Estimate	Standard Deviation X 10^{+2}
C_{11}	0.01893	0.04340	0.02008	0.06121
C_{12}	-0.00391	0.02691	-0.00280	0.05140
C_{22}	0.00502	0.03326	0.03653	2.219
P_1	0.6345	0.8860	0.6266	0.9004
P_2	-0.2943	3.407	-0.9026	7.045
P_3	-0.1762	3.209	0.2748	8.853
P_4	0.8818	2.403	0.5045	5.557

sets. The behavior of the models is shown graphically in Figures 4 and 5 along with the data for comparison. It is felt that a better model was obtained for data set FD21-2 than for FD21-3 and this is confirmed by the standard deviation of the residuals which was 0.00297 for data set FD21-2 and 0.00594 for data set FD21-3. The poorer fit of data set FD21-3 is due to the inconsistent behavior in this data set. This inconsistency can be best seen by looking at Figure 3 which shows the fill direction stress-strain behavior for data set FD21-3. If we look at the behavior for a constant value of T_1 , the fill stress, we see that the fill strain behaves erratically as the stress ratio increases. Since increases in the stress ratio for constant fill stress requires increases in warp stress, we expect on physical grounds that the fill strain will decrease monotonically as the stress ratio increases with the fill stress held constant. This behavior is not seen in the data in Figure 3; in fact, the strain data has neither a monotonically increasing nor decreasing behavior with the stress ratio, and while model 4 could emulate either of these behaviors, it does not have the capability to emulate the behavior shown in the figure. As a result we get a poorer model. One might question the use of data with such inconsistencies and in response we just point out that this was a set of data obtained by the current state of the art in testing, and it is included here merely to point out some of the difficulties that must be taken care of in modeling fabric stress-strain data. The data set FD21-2 does not have the inconsistent behavior described above and thus we get a better fit of that data. The inconsistencies discussed here are not felt to be fundamental errors in the data but represent difficulties in the test procedure. In particular, it is difficult to be sure that two tests that are supposed to be identical are in fact identical. This difficulty is felt to be associated with the start-up of the test, in making sure that both the fill and warp stresses are zero at the start and that the stress ratio undergoes the same transient behavior in the initial part of the test, as it seeks its assigned value. During this initial period, fabrics undergo substantial deformation due to crimp interchange with small changes in the stress state. As a result, small variations in the application of stress can cause significant changes in the resulting strain. The model does appear to be weak in predicting all the strain contractions that occur at low stress levels. This can be seen in Figure 4 for the warp behavior. The model is able to predict the contraction for the 6/1 stress ratio but not that for the 2/1 ratio.

The standard deviations of the parameter estimates given in Table 1 are also a statistical measure of the goodness of the model. We can claim that the parameters are not zero, that is, they should be in the model, if the absolute values of the estimates are greater than twice their standard deviations. This criterion is satisfied for all parameters with the model generated for data set FD21-2 but not for the model associated with data set FD21-3 where we cannot claim according to the above criterion that the parameter C_{22} is nonzero. The parameter could be dropped with the model to create a new model, but because of inconsistencies in the data this was not done.

Model 4 is thought to be satisfactory for modeling fabric stress-strain behavior of which data set FD21-2 is typical.

The next model we describe, which is referred to as model 5, is also a power law model and it is somewhat more flexible than model 4 but this increased flexibility is at the cost of the symmetry property stated as equation (3). Model 5 has the form

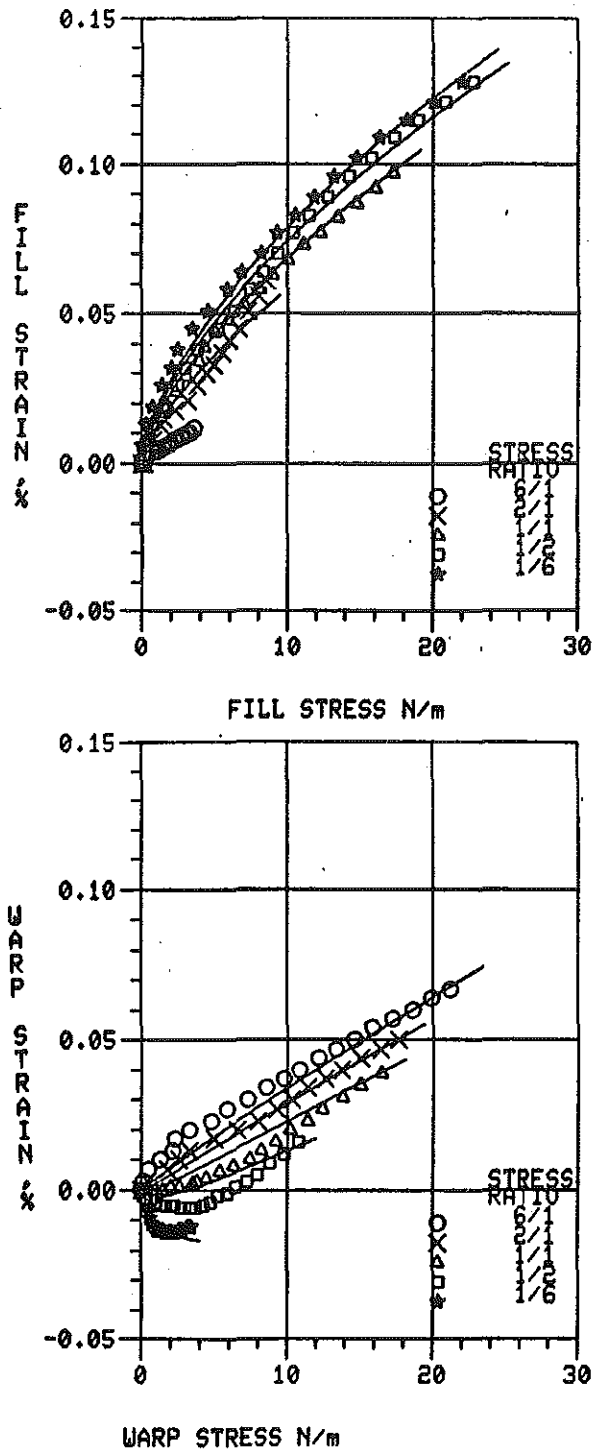


Figure 4. Behavior of model 4 based on data set FD21-2 and comparison with data.

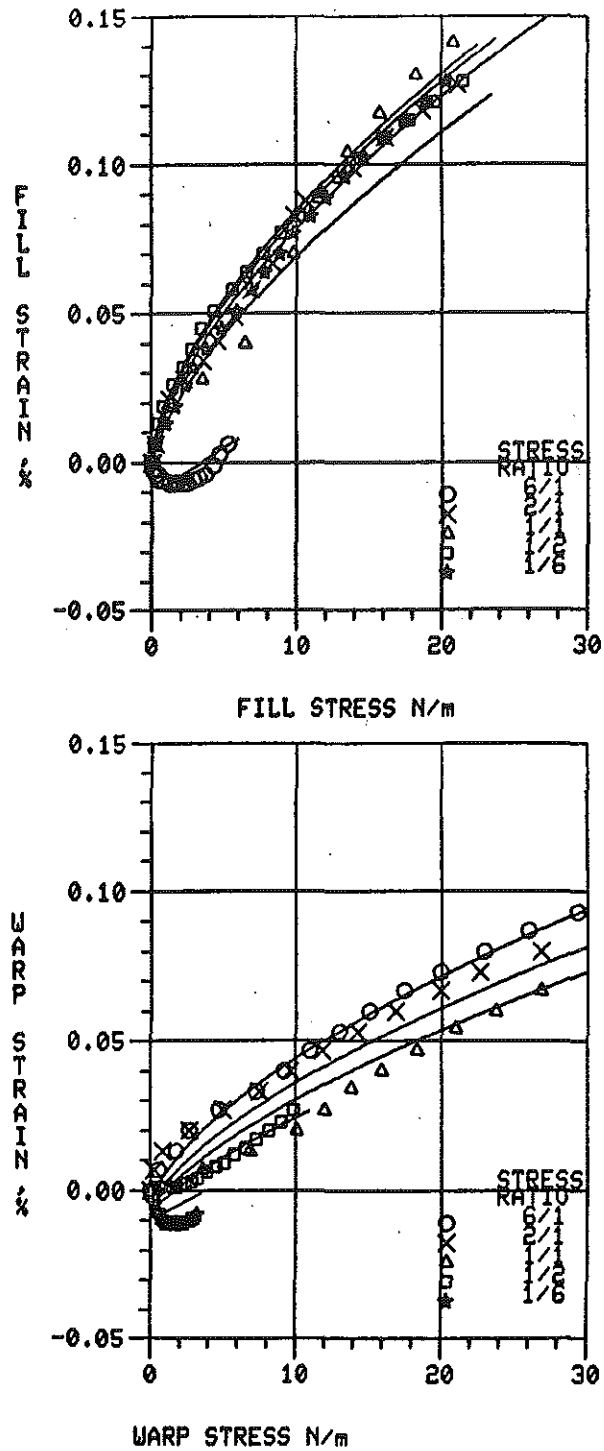


Figure 5. Behavior of model 4 based on data set FD21-3 and comparison with data.

$$\begin{aligned}
E_1 &= C_{11} T_1^{P_1} + C_{12} T_2^{P_2} \\
E_2 &= C_{21} T_1^{P_3} + C_{22} T_2^{P_4}
\end{aligned} \tag{5}$$

This model can also be reduced to a linear model by setting the exponents P_i equal to unity. Listed in Table 2 are the estimates of the function parameters and their standard deviations obtained from the nonlinear least squares fitting of each of the data sets.

As with model 4 these two sets of parameter estimates reflect the differences in the data sets. In Figures 6 and 7 the behavior of these models is shown graphically along with a comparison with the data from which the models were generated. The increased flexibility of the model, by addition of one parameter and removal of the symmetry constraint, did not provide significant improvement in the models as measured by the standard deviation of the residuals. This parameter was 0.00257 for data set FD21-2 and 0.00779 for FD21-3. Thus model 5 gave a poorer fit than model 4 for data set FD21-3. Model 5 is much like model 4 with regard to its ability to emulate the inconsistent behavior of data set FD21-3 and to predict the strain contractions at high stress ratios. This inability of the models to emulate the inconsistencies of the data is not thought to be a deficiency of the models. Regarding the goodness of fit criterion, which requires that the absolute values of the parameter estimates be twice their standard deviations for the parameters to be declared nonzero, model 5 satisfactorily passes for the models for both data sets. However, here as with model 4, the standard deviations for the models associated with data set FD21-2 are generally smaller than those for the models associated with data set FD21-3.

We now turn to the final model to be described here which is designated model 11. This model has more flexibility than models 4 or 5 and does not have the symmetry property of equation (3). This model was motivated by the inability of the previous models to predict the strain contractions at high stress ratios. This suggested the need for a variable power law model in which the exponents are not constants but functions of the stress level. As a result we constructed the following model in which the exponents are linear functions of the stresses.

$$\begin{aligned}
E_1 &= C_{11} T_1^{P_{10} + P_{11} T_1 + P_{12} T_2} + C_{12} T_2^{P_{20} + P_{21} T_1 + P_{22} T_2} \\
E_2 &= C_{21} T_1^{P_{30} + P_{31} T_1 + P_{32} T_2} + C_{22} T_2^{P_{40} + P_{41} T_1 + P_{42} T_2}
\end{aligned} \tag{6}$$

The estimates of the function parameters obtained from the least squares fit of the two data sets are presented in Table 3 along with the associated standard deviations. The increased flexibility of this model provides a better fit to both data sets with standard deviations of the residuals of 0.00206 for data set FD21-2 and 0.00401 for data set FD21-3. The behavior of this model along with a comparison with the fitted data are presented in Figures 8 and 9 for data sets FD21-2 and FD21-3, respectively. Examination of these figures reveals that model 11 does better in predicting the fabric strain contractions at high stress ratios and in doing so has a smoother character compared with models 4 and 5, which show some rather sharp changes in slope in predicting these contractions.

Table 2. Parameter Estimates for Model 5

Parameter	Data Set FD21-2		Data Set FD21-3	
	Parameter Estimate	Standard Deviation X 10^{+2}	Parameter Estimate	Standard Deviation X 10^{+2}
C_{11}	0.02241	0.1123	0.02956	0.2646
C_{12}	-0.009002	0.1274	-0.02152	0.3808
C_{21}	-0.009005	0.0911	-0.00987	0.2973
C_{22}	0.009086	0.0858	0.01360	0.2410
P_1	0.6106	1.411	0.5720	2.144
P_2	0.4374	3.569	0.2403	3.454
P_3	0.4748	2.503	0.4426	7.674
P_4	0.7286	2.920	0.6304	4.535

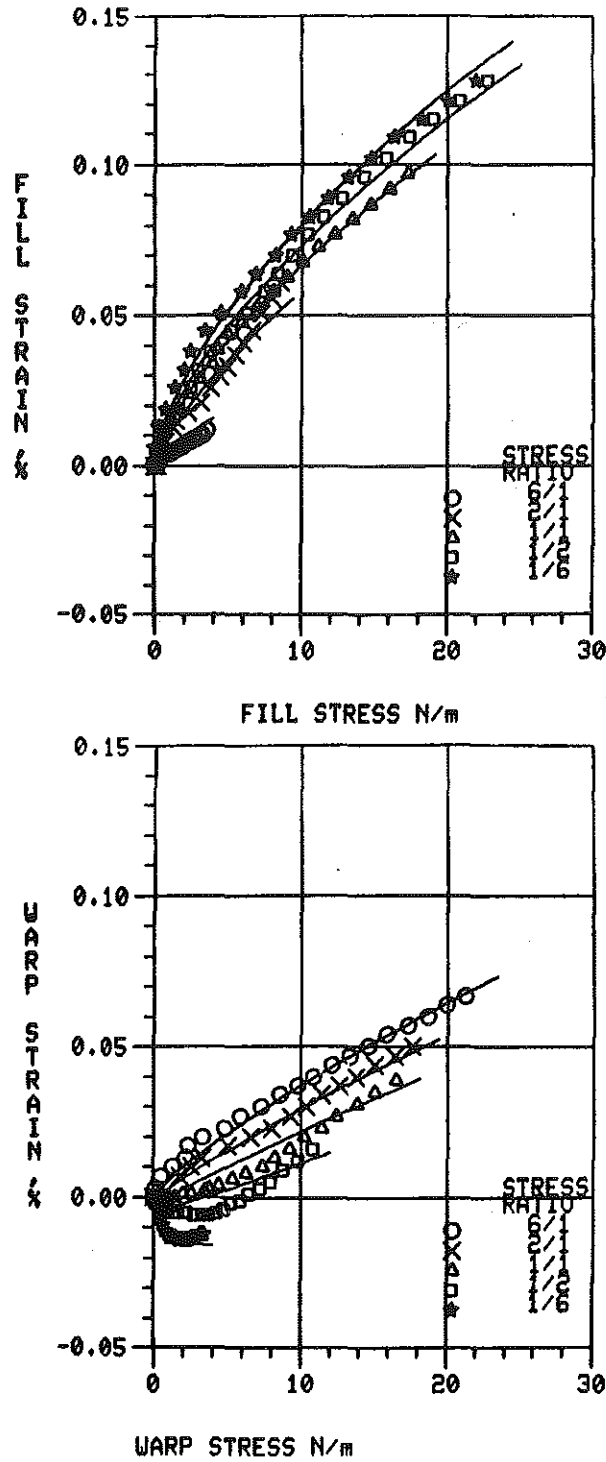


Figure 6. Behavior of model 5 based on data set FD21-2 and comparison with data.

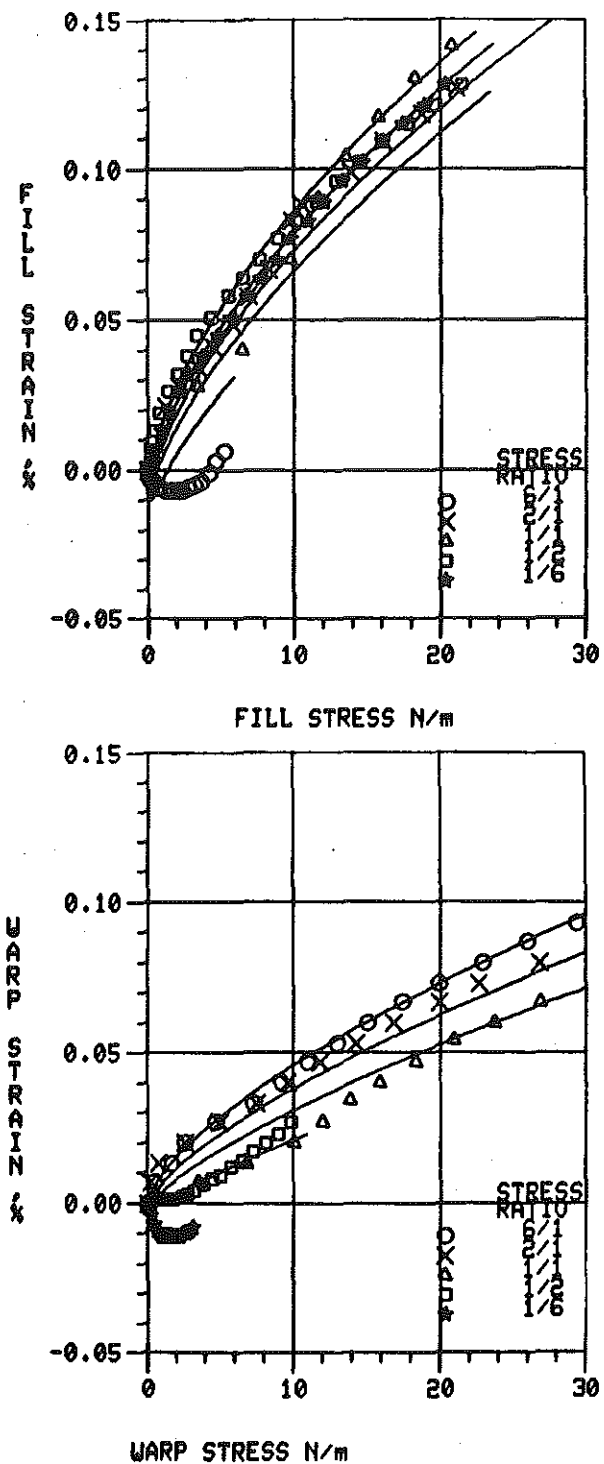


Figure 7. Behavior of model 5 based on data set FD21-3 and comparison with data.

Table 3. Parameter Estimates for Model 11

Parameter	Data Set FD21-2		Data Set FD21-3	
	Parameter Estimate	Standard Deviation X 10^{+2}	Parameter Estimate	Standard Deviation X 10^{+2}
C _{1 1}	0.02300	0.1839	0.01698	0.1264
C _{1 2}	-0.00846	0.1867	-0.002897	0.1263
C _{2 1}	-0.00845	0.1513	-0.009361	0.2450
C _{2 2}	0.01072	0.1501	0.01457	0.2600
P _{1 0}	0.5884	3.156	0.6940	2.894
P _{1 1}	0.00009	0.05275	-0.001101	0.03130
P _{1 2}	-0.00265	0.0763	0.000236	0.02201
P _{2 0}	0.5584	8.407	1.122	19.39
P _{2 1}	-0.02672	0.7091	-0.1436	2.832
P _{2 2}	-0.001615	0.1003	0.01334	0.4102
P _{3 0}	0.6430	7.969	0.5495	9.008
P _{3 1}	-0.007826	0.1978	-0.004578	0.2334
P _{3 2}	-0.004408	0.6899	-0.004609	0.2076
P _{4 0}	0.6330	5.703	0.5990	6.229
P _{4 1}	-0.001187	2.956	-0.001704	0.03984
P _{4 2}	0.002527	0.1020	0.000408	0.16058

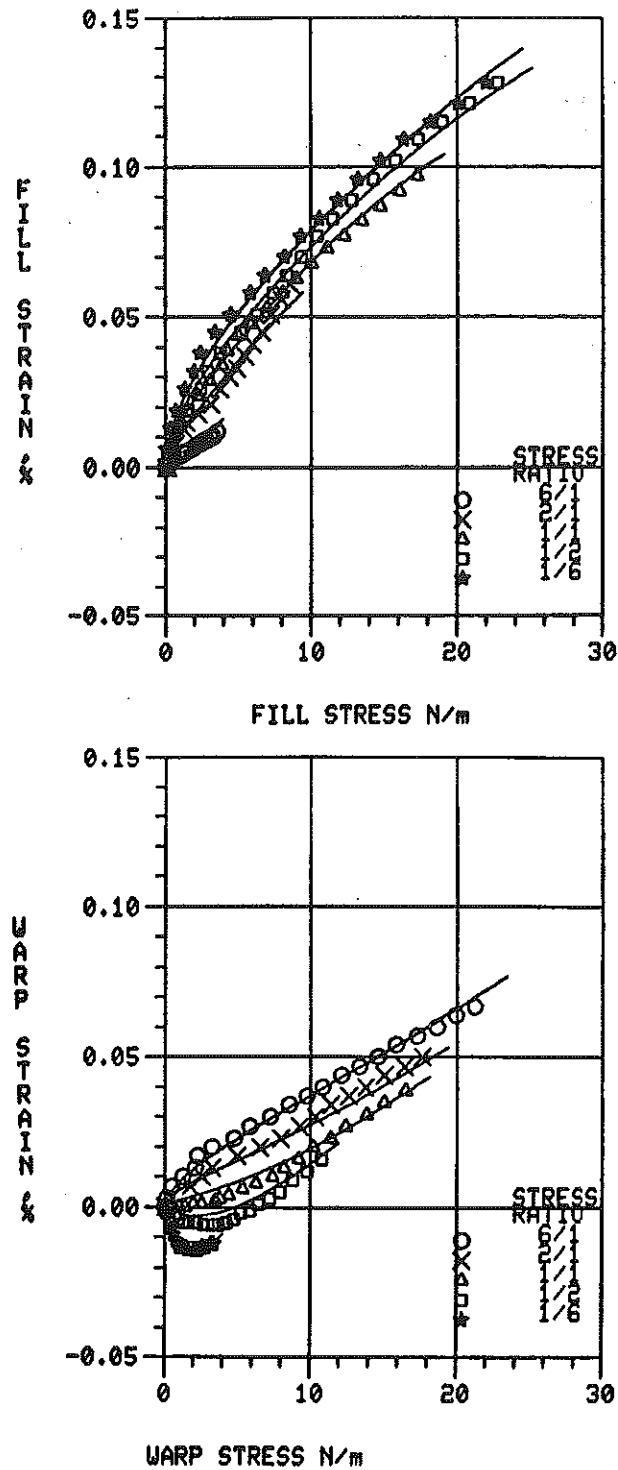


Figure 8. Behavior of model 11 based on data set FD21-2 and comparison with data.

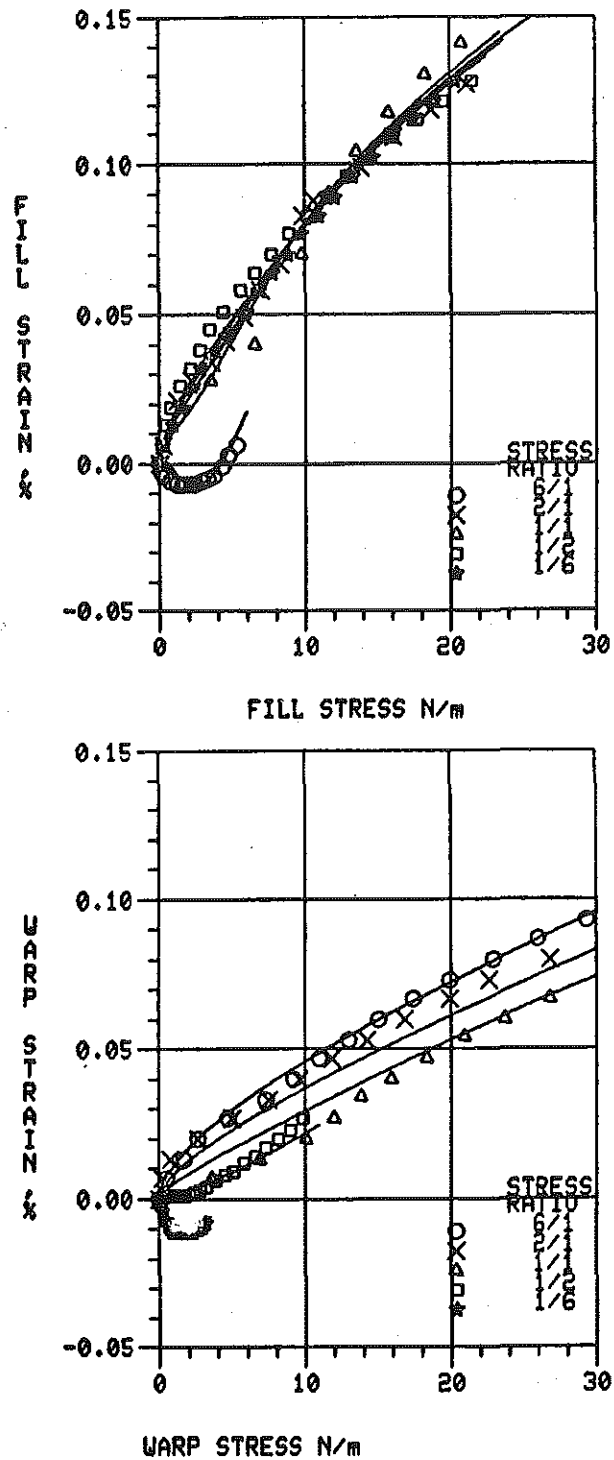


Figure 9. Behavior of model 11 based on data set FD21-3 and comparison with data.

However, the model does appear to have more flexibility than needed in the sense that it is possible to predict the inconsistencies that are present in data set FD21-3. That is, with model 11 it is possible with a constant fill stress to have non-monotonic changes in fill strain with monotonic increase in the stress ratio, while on physical grounds one expects monotonic decreases in the fill strain with increasing stress ratio. This raises questions as to whether the variability of the exponents is needed to predict the behavior of data set FD21-2 which does not have this inconsistent behavior. To answer such questions we examine the degree of variability of the exponents in the model 11 constructed for data set FD21-2. As can be seen from the results plotted in Figure 10, three of the exponents show rather small variability over the stress range considered in this work. The fourth exponent, that of T_2 in the expression for E_1 , varies over a range from near zero to something slightly more than one-half, and this is considered an important variation. Of the three exponents which show small variation, the one with the largest variation is the other term specifying the Poisson's Ratio effect, that is, the exponent of T_1 in the expression for E_2 .

Model 11 provides a better fit of the data which may be beneficial, especially if one wants to do further analysis of the data such as creating an incremental model of the form (2) where derivatives are necessary. However, care should be taken to make sure the data is consistent in the sense already discussed. The usefulness of model 11 may be questioned on the grounds of its lack of symmetry specified by equation (3) for constructing an incremental model, but this difficulty could be overcome by imposing a symmetry by some arbitrary procedure such as given by the following incremental law

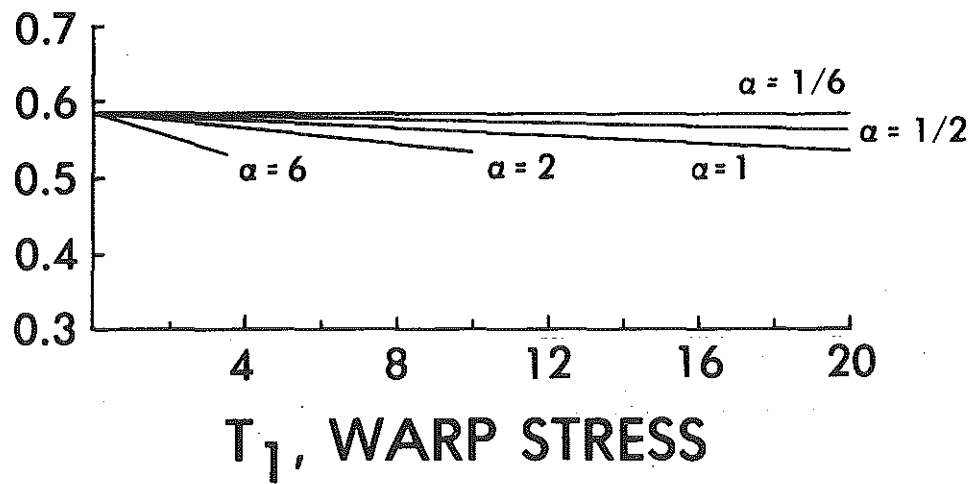
$$\begin{aligned}\Delta E_1 &= C_{11}\Delta T_1 + C_{12}\Delta T_2 \\ \Delta E_2 &= C_{12}\Delta T_1 + C_{22}\Delta T_2\end{aligned}\tag{7}$$

where

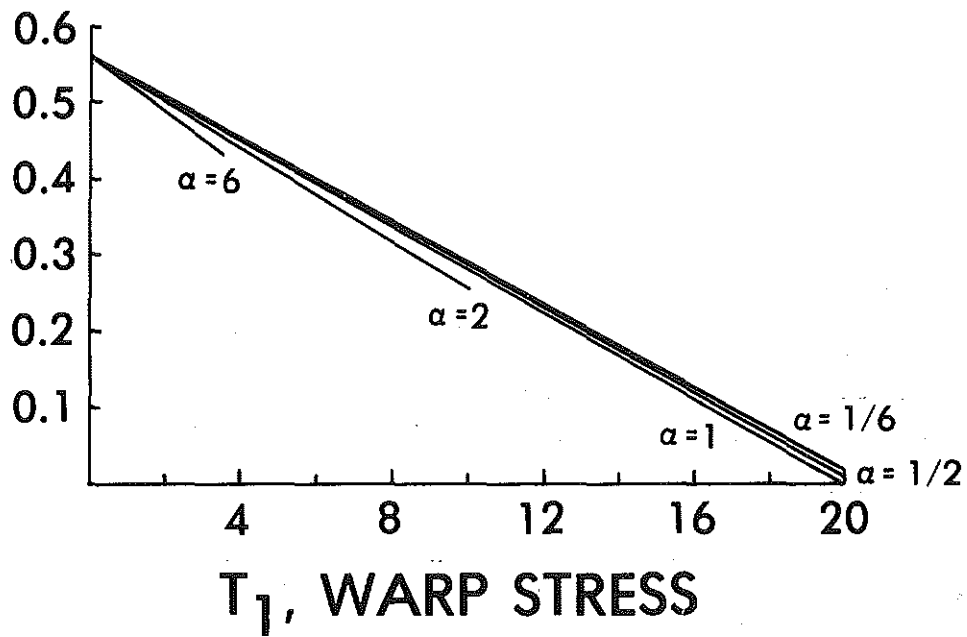
$$\begin{aligned}C_{11} &= \frac{\partial E_1}{\partial T_1} \\ C_{22} &= \frac{\partial E_2}{\partial T_2} \\ C_{12} &= \frac{1}{2} \left[\frac{\partial E_1}{\partial T_2} + \frac{\partial E_2}{\partial T_1} \right]\end{aligned}\tag{8}$$

Another alternative would be to perform a constrained least squares. That is, to minimize the sum of the squared residuals subject to the constraint on the partial derivatives given in (3). Doing this requires some software capability not included in the NL2SOL program used in this work.

An examination of the standard deviations of the parameter estimates for model 11 given in Table 3 reveals that it is not possible to claim that all of the parameters are nonzero.

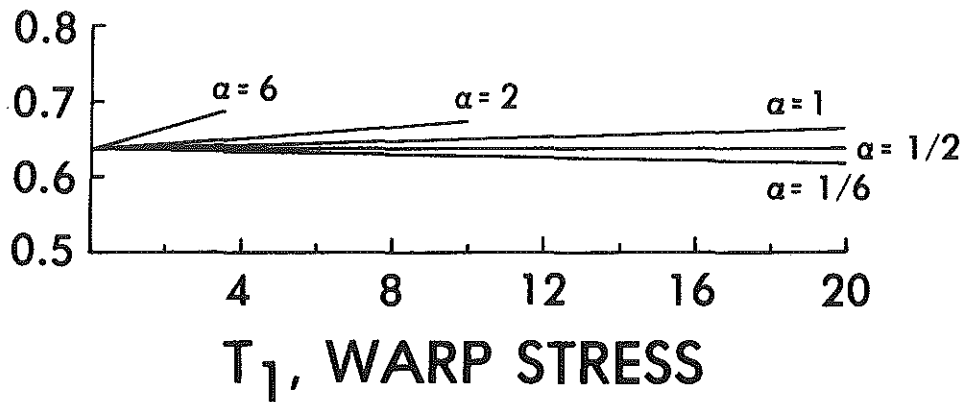


(a) EXPONENT OF T_1 IN EXPRESSION FOR E_1

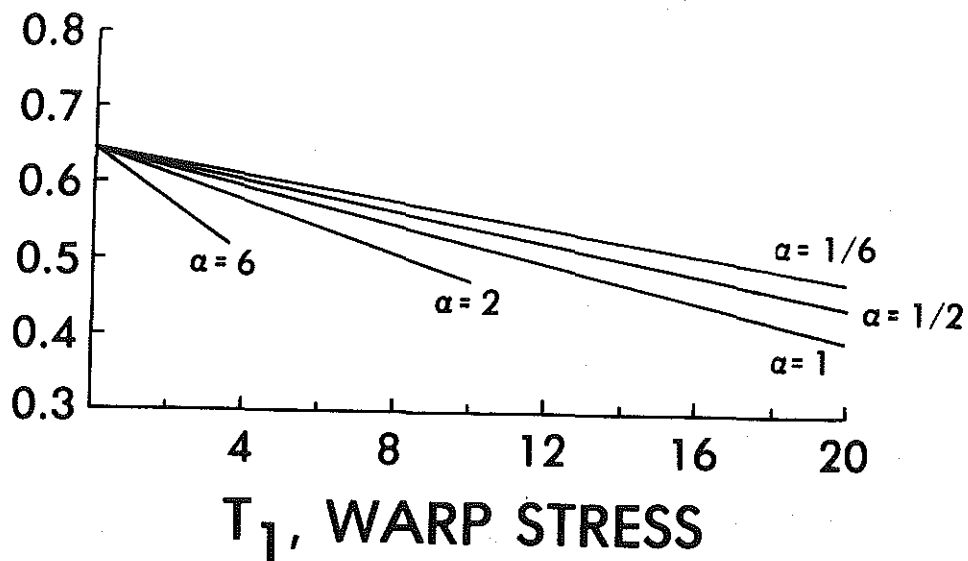


(b) EXPONENT OF T_2 IN EXPRESSION FOR E_1

Figure 10. Behavior of variable exponents of model 11.



(c) EXPONENT OF T_2 IN THE EXPRESSION FOR E_2



(d) EXPONENT OF T_1 IN THE EXPRESSION FOR E_2

Figure 10. Concluded

This is the result of the estimates being less than twice their standard deviations. For the model 11 associated with data set FD21-2, which we will examine more closely, the parameters P_{11} , P_{22} , P_{32} , and P_{41} cannot in a statistical sense be declared nonzero. This suggests a new model in which these parameters are not included, namely

$$E_1 = C_{11}T_1 P_{10} + P_{12}T_2 + C_{12}T_2 P_{20} + P_{21}T_1$$

$$E_2 = C_{21}T_1 P_{30} + P_{31}T_1 + C_{22}T_2 P_{40} + P_{42}T_2$$

The least squares analysis of data set FD21-2 with this model resulted in the parameter estimates and standard deviations given in Table 4. The standard deviation of the residuals from this analysis was 0.00214 and this compared with the 0.404×10^{-3} for the complete model 11 indicates that the reduced model gives a slightly poorer fit but it is a more economical model in that it contains four fewer parameters. All of these parameters satisfy the statistical criterion for being nonzero. The behavior of this reduced model 11 is shown graphically in Figure 11 along with the data from data set FD21-2 used to generate the model. This reduction or modification of the model should not be generalized as being suitable for all data sets and all fabrics. In fact, if we had more data or the same amount of data at different values of the stress, T_1 and T_2 , we might be able to determine estimates for all 16 parameters of model 11 which would be nonzero in the statistical sense.

CONCLUDING REMARKS

A study of the nonlinear least squares modeling of the biaxial stress-strain behavior of fabrics has been completed and it was found that power law models gave satisfactory representation of the behavior of a typical fabric.

This document reports research undertaken at the US Army Natick Research and Development Command and has been assigned No. NATICK/TR-82/009 in the series of reports approved for publication.

**Table 4. Parameter Estimates for the Reduced Model 11
and Data Set FD21-2**

Parameter	Parameter Estimate	Standard Deviation X 10⁺²
C _{1 1}	0.0224	0.1026
C _{1 2}	-0.008197	0.1106
C _{2 1}	-0.008318	0.1427
C _{2 2}	0.01028	0.1476
P _{1 0}	0.5998	1.265
P _{1 2}	0.004472	0.1519
P _{2 0}	0.4906	1.153
P _{2 1}	0.00946	0.4180
P _{3 0}	0.6316	6.619
P _{3 1}	-0.007748	0.1460
P _{4 0}	0.6563	5.937
P _{4 2}	0.002203	0.1101

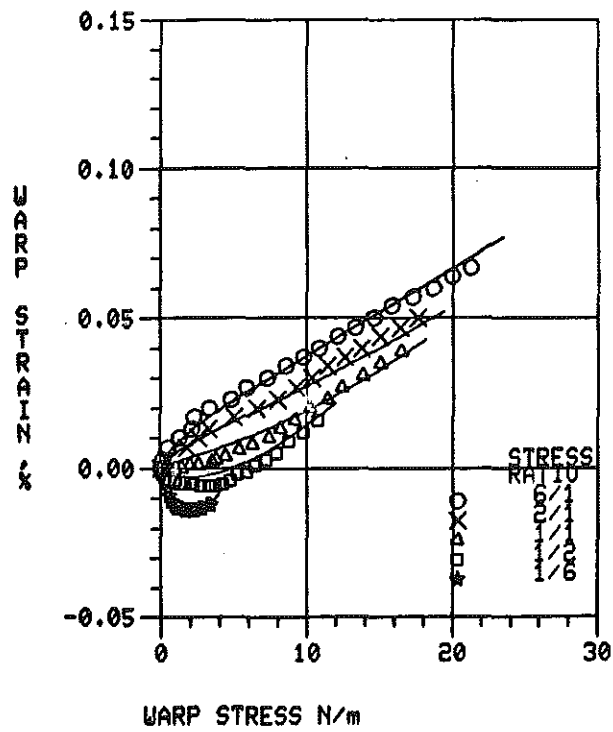
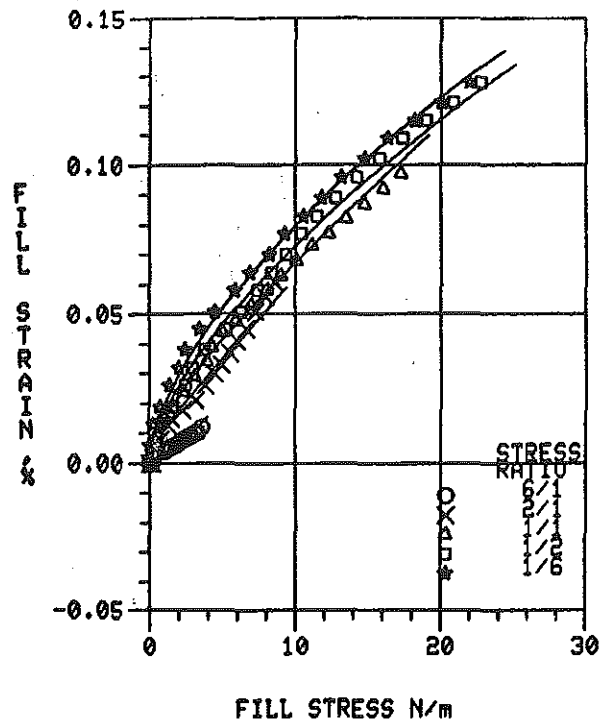


Figure 11. Behavior of the reduced model 11 based on data set FD21-2 and comparison with data.

APPENDIX A
BIAXIAL STRESS-STRAIN DATA

APPENDIX A

BIAXIAL STRESS-STRAIN DATA DATA SET FD21-2

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio-6 (S102-10)			
.000	.000	.000	.000
.076	.000	.154	.003
.152	.000	.541	.007
.306	.003	1.315	.010
.461	.003	2.205	.013
.501	.004	2.340	.017
.619	.004	3.328	.020
.854	.004	4.838	.023
1.091	.004	5.882	.027
1.252	.005	7.355	.030
1.492	.005	8.632	.034
1.733	.006	9.794	.037
1.859	.006	10.899	.040
2.064	.007	12.158	.044
2.272	.008	13.419	.047
2.480	.008	14.679	.050
2.650	.009	15.920	.054
2.902	.009	17.316	.057
3.075	.010	18.674	.060
3.329	.010	19.953	.064
3.590	.012	21.237	.067

Fill Direction		Stress Ratio—2 (S102—7)	Warp Direction	
Stress	Strain		Stress	Strain
.000	.000		.000	.000
.227	.000		.462	.003
.843	.008		1.817	.007
1.236	.013		2.597	.010
1.554	.015		3.336	.013
2.347	.018		5.012	.017
3.149	.021		6.577	.020
3.808	.026		8.071	.023
4.476	.030		9.339	.027
4.984	.033		10.406	.030
5.505	.037		11.482	.034
6.036	.041		12.681	.037
6.652	.045		13.806	.040
7.290	.051		15.123	.044
8.016	.056		16.483	.047
8.586	.061		17.707	.050

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio—1 (S101—4)			
.000	.000	.000	.000
.266	.005	.286	.000
.593	.010	.575	.000
1.115	.015	1.011	.000
1.638	.019	1.451	.000
2.355	.024	2.042	.001
3.074	.029	2.639	.001
3.997	.034	3.537	.002
4.329	.039	3.854	.003
5.052	.044	4.472	.004
5.987	.048	5.403	.006
7.046	.053	6.343	.008
8.126	.058	7.454	.010
9.066	.063	8.274	.013
10.087	.068	9.265	.016
11.181	.073	10.275	.020
12.351	.077	11.452	.023
13.520	.082	12.495	.027
14.854	.087	13.880	.031
16.115	.092	15.118	.035
17.345	.097	16.546	.039

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio—1/2 (S101—9)			
.000	.000	.000	.000
.317	.006	.143	— .002
.793	.013	.361	— .002
1.586	.019	.726	— .003
2.465	.026	1.021	— .004
2.975	.032	1.320	— .005
3.828	.038	1.696	— .005
5.244	.045	2.225	— .005
6.352	.051	2.761	— .006
7.460	.058	3.303	— .006
8.411	.064	3.776	— .006
9.363	.070	4.257	— .005
10.441	.077	4.746	— .004
11.523	.083	5.244	— .002
12.799	.089	5.904	— .001
14.290	.096	6.498	.001
15.857	.102	7.260	.003
17.392	.109	7.956	.005
19.009	.115	8.830	.009
20.918	.121	9.805	.012
22.833	.128	10.797	.016

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio—1/6 (S101—10)			
.000	.000	.000	.000
.018	.006	.036	— .001
.335	.013	.180	— .002
.758	.019	.291	— .002
1.408	.026	.365	— .003
2.058	.032	.440	— .005
2.461	.038	.515	— .007
3.498	.045	.628	— .009
4.534	.051	.742	— .011
5.868	.058	.970	— .012
6.922	.064	1.124	— .013
8.221	.070	1.281	— .013
9.293	.077	1.479	— .013
10.610	.083	1.678	— .014
11.928	.089	1.879	— .014
13.245	.096	2.083	— .014
14.826	.102	2.328	— .014
16.409	.109	2.577	— .013
18.248	.115	2.853	— .013
20.101	.121	3.163	— .012
22.053	.128	3.422	— .012

BIAXIAL STRESS-STRAIN DATA
DATA SET FD21-3

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio-6 (S102-20)			
.000	.000	.000	.000
.153	-.003	.537	.007
.346	-.004	1.784	.013
.465	-.004	2.648	.020
.778	-.006	4.757	.027
1.252	-.007	7.288	.033
1.574	-.007	9.281	.040
1.822	-.007	11.008	.047
2.152	-.007	13.003	.053
2.487	-.007	15.112	.060
2.868	-.006	17.474	.067
3.337	-.005	19.991	.073
3.772	-.004	22.950	.080
4.304	-.001	26.071	.087
4.741	.003	29.378	.093
5.315	.006	33.094	.100

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio—2 (S102—21)			
.000	.000	.000	.000
.077	.000	.269	.007
.311	.006	.846	.013
1.270	.021	2.710	.020
2.251	.028	5.046	.027
3.583	.034	7.556	.033
4.623	.041	9.692	.040
5.857	.049	11.926	.047
6.963	.058	14.331	.053
8.363	.067	16.889	.060
9.865	.083	19.995	.067
10.679	.088	22.702	.073
13.933	.099	26.915	.080
16.168	.109	30.843	.087
18.725	.118	35.162	.093
21.160	.127	39.653	.100

Fill Direction		Stress Ratio—1 (S102—23)	Warp Direction	
Stress	Strain		Stress	Strain
.000	.000		.000	.000
3.522	.028		3.617	.007
6.516	.040		6.865	.013
9.793	.070		10.138	.020
11.764	.090		12.037	.027
13.583	.104		13.901	.034
15.808	.117		15.985	.040
18.289	.130		18.390	.047
20.818	.141		20.930	.054
23.812	.153		23.737	.060
27.060	.163		26.906	.067
30.740	.172		30.408	.074
34.747	.182		34.340	.081
39.343	.194		38.649	.087
44.485	.206		43.392	.094
49.477	.211		48.021	.101
54.757	.221		52.905	.107
60.464	.237		58.237	.113

Fill Direction		Stress Ratio—1/2 (S105—5)	Warp Direction	
Stress	Strain		Stress	Strain
.000	.000		.000	.000
.285	.006		.071	.000
.480	.013		.214	.000
.836	.019		.430	.001
1.442	.026		.649	.001
2.208	.032		.944	.001
2.849	.038		1.242	.001
3.454	.045		1.470	.001
4.380	.051		1.849	.001
5.628	.058		2.383	.002
6.593	.064		2.849	.003
7.773	.070		3.247	.004
8.938	.077		3.881	.006
10.230	.083		4.525	.008
11.612	.089		5.100	.009
12.949	.096		5.846	.012
14.536	.102		6.599	.014
16.093	.109		7.289	.017
17.855	.115		8.154	.020
19.622	.121		9.034	.023
21.519	.128		9.928	.027

Fill Direction		Warp Direction	
Stress	Strain	Stress	Strain
Stress Ratio—1/6 (S105—6)			
.000	.000	.000	.000
.285	.006	.071	— .001
.980	.013	.213	— .002
1.639	.019	.285	— .004
2.333	.026	.430	— .007
2.868	.032	.504	— .007
3.811	.038	.651	— .009
4.879	.045	.800	— .010
5.947	.051	.950	— .011
6.890	.058	1.102	— .011
7.834	.064	1.257	— .011
8.903	.070	1.413	— .011
9.846	.077	1.571	— .011
11.006	.083	1.656	— .011
12.074	.089	1.817	— .011
13.427	.069	2.056	— .011
14.781	.102	2.299	— .011
16.136	.109	2.545	— .010
17.456	.115	2.716	— .010
18.937	.121	2.968	— .009
20.383	.128	3.145	— .008

APPENDIX B
PROCEDURE FOR EXECUTION OF THE
MODELING CODE

APPENDIX B

PROCEDURE FOR EXECUTION OF THE MODELING CODE

Here we give the details for the execution of the modeling codes on the UNIVAC 1106 computer at NLABS. We include both the single and double precision versions. A run stream for the single (double) precision version is given in Table 1 (2). Statement 1 assigns the tape on which the modeling codes are stored and statement 2 positions the tape so these codes can be read into the files assigned by statement 3 (3-5). The read-in of the codes is accomplished by statement 4 (6-8). Upon completion of these read-in statements the file named E. contains the single precision codes including the NL2SOL subroutines in relocatable form; the main program and associated subroutines to carry out the data handling, I/O functions and computation peculiar to the fabric modeling problem; and the fabric stress-strain data used to generate the models. This is the only file needed for the single precision version. The file DPE. contains the double precision versions of the main program and associated subroutines to carry out the data handling, I/O functions and computations peculiar to the fabric modeling problem and some control data for NL2SOL that are peculiar to the double precision version. The file NLDBL contains the double precision versions of the NL2SOL subroutines in relocatable form. The file E is needed for the execution of the double precision version because the stress-strain data is stored there. Statements 5-9 (9-13) call the FORTRAN compiler for the main program and subroutines peculiar to the fabric modeling and statements 10-14 (14-18) perform the collection operation resulting in an absolute executable program in TPF .B. The fabric stress-strain data is put in the temporary file 2. by statements 15-18 (19-22). The modeling is executed and data entered by the last four statements. The first four numbers in statement 20 (24) define the size of the plots and the last one can be used for weighting certain of the data points. The fabric stress-strain data used in the least squares modeling is entered by statement 21 (25). This is the same data as is entered in file 2. only in a different format. A guess at the solution which is used in starting the nonlinear least squares solution process is entered by statement 22 (26). There are several additional data inputs but the program requests these inputs when required. One of these requests is for the parameters that control NL2SOL and the response to this request should be @ ADD E.PARMS for the single precision and @ ADD DPE.PARMS for the double precision version. The main program is written to run on the UNIVAC 1106 computer and the plotted output is done on the TEKTRONIX Advanced Graphing II software.

RUNSTREAM FOR SINGLE PRECISION VERSION

```
1 @ASG,T TP,6C9,1964
2 @MOVE TP,11
3 @ASG,T E.
4 @COPIN TP.,E.
5 @FOR,S E.M,M
6 @FOR,S E.PFD3,PFD3
7 @FOR,S E.CALCR3,CR
8 @FOR,S E.EPS1-4,E1
9 @FOR,S E.EPS2-4,E2
10 @MAP,I A,B
11 IN TPF$.M
12 IN NLDBL.
13 LIB DAO*TEX.
14 END
15 @ASG,T 2.
16 @DATA,I 2.
17 @ADD,D E.FD21-2
18 @END
19 @XQT B
20 25.0,0.25,-0.025,0.15,1.0
21 @ADD E.EXDA21-2
22 @ADD E.DAIN4
```

RUNSTREAM FOR DOUBLE PRECISION VERSION

```

1  @ASG,T  TP,6C9,1964
2  @MOVE  TP,11
3  @ASG,T  E.
4  @ASG,T  DPE.
5  @ASG,T  NLDBL.
6  @COPIN  TP.,E.
7  @COPIN  TP.,DPE.
8  @COPIN  TP.,NLDBL.
9  @FOR,S  DPE.M,M
10 @FOR,S  DPE.PFD3,PFD3
11 @FOR,S  DPE.CALCR3,CR
12 @FOR,S  DPE.EPS1-4,E1
13 @FOR,S  DPE.EPS2-4,E2
14 @MAP,I  A,B
15   IN  TPF$.M
16   IN  NLDBL.
17  LIB  DAO*TEX.
18  END
19 @ASG,T  2.
20 @DATA,I  2.
21 @ADD,D  E.FD21-2
22 @END
23 @XQT  B
24 25.0,0.25,-0.025,0.15,1.0
25 @ADD  E.EXDA21-2
26 @ADD  E.DAIN4

```